Philosophy 405: Knowledge, Truth and Mathematics
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John Locke on Mathematics from *Essay Concerning Human Understanding*

§I. On Abstraction

Book III, Chapter 1: Of Words or Language in General

1. *Man fitted to form articulate sounds*. God, having designed man for a sociable creature, made him not only with an inclination, and under a necessity to have fellowship with those of his own kind, but furnished him also with language, which was to be the great instrument and common tie of society. Man, therefore, had by nature his organs so fashioned, as to be fit to frame articulate sounds, which we call words. But this was not enough to produce language; for parrots, and several other birds, will be taught to make articulate sounds distinct enough, which yet by no means are capable of language.

2. *To use these sounds as signs of ideas*. Besides articulate sounds, therefore, it was further necessary that he should be able to use these sounds as signs of internal conceptions; and to make them stand as marks for the ideas within his own mind, whereby they might be made known to others, and the thoughts of men's minds be conveyed from one to another.

3. *To make them general signs*. But neither was this sufficient to make words so useful as they ought to be. It is not enough for the perfection of language, that sounds can be made signs of ideas, unless those signs can be so made use of as to comprehend several particular things: for the multiplication of words would have perplexed their use, had every particular thing need of a distinct name to be signified by. To remedy this inconvenience, language had yet a further improvement in the use of general terms, whereby one word was made to mark a multitude of particular existences: which advantageous use of sounds was obtained only by the difference of the ideas they were made signs of: those names becoming general, which are made to stand for general ideas, and those remaining particular, where the ideas they are used for are particular...

Chapter 2: Of the Signification of Words

1. *Words are sensible signs, necessary for communication of ideas*. Man, though he have great variety of thoughts, and such from which others as well as himself might receive profit and delight; yet they are all within his own breast, invisible and hidden from others, nor can of themselves be made to appear. The comfort and advantage of society not being to be had without communication of thoughts, it was necessary that man should find out some external sensible signs, whereof those invisible ideas, which his thoughts are made up of, might be made known to others. For this purpose nothing was so fit, either for plenty or quickness, as those articulate sounds, which with so much ease and variety he found himself able to make. Thus we may conceive how words, which were by nature so well adapted to that purpose, came to be made use of by men as the signs of their ideas; not by any natural connexion that there is between particular articulate sounds and certain ideas, for then there would be but one language amongst all men; but by a voluntary imposition, whereby such a word is made arbitrarily the mark of such an idea. The use, then, of words, is to be sensible marks of ideas; and the ideas they stand for are their proper and immediate signification.

2. *Words, in their immediate signification, are the sensible signs of his ideas who uses them*. The use men have of these marks being either to record their own thoughts, for the assistance of their own memory or, as it were, to bring out their ideas, and lay them before the view of others: words, in their primary or immediate signification, stand for nothing but the ideas in the mind of him that uses them, how imperfectly soever or carelessly those ideas are collected from the things which they are supposed to represent. When a man speaks to another, it is that he may be understood: and the end of speech is, that those sounds, as marks, may make known his ideas to the hearer. That then which words are the marks of are the ideas of the speaker: nor can any one apply them as marks, immediately, to anything else but the ideas that he himself hath: for this would be to make them signs of his own conceptions, and yet apply them to other ideas; which would be to make them signs and not signs of his ideas at the same time, and so in effect to have no signification at all. Words being voluntary signs, they cannot be voluntary signs imposed by him on things he knows not. That would be to make them signs of nothing, sounds without signification. A man cannot make his words the signs either of qualities in things, or of conceptions in the mind of another, whereof he has none in his own. Till he has some ideas of his own, he cannot suppose them to correspond with the conceptions of another man; nor can he use any signs for them of another man; nor can he use any

signs for them: for thus they would be the signs of he knows not what, which is in truth to be the signs of nothing. But when he represents to himself other men's ideas by some of his own, if he consent to give them the same names that other men do, it is still to his own ideas; to ideas that he has, and not to ideas that he has not.

3. *Examples of this*. This is so necessary in the use of language, that in this respect the knowing and the ignorant, the learned and the unlearned, use the words they speak (with any meaning) all alike. They, in every man's mouth, stand for the ideas he has, and which he would express by them. A child having taken notice of nothing in the metal he hears called gold, but the bright shining yellow colour, he applies the word gold only to his own idea of that colour, and nothing else; and therefore calls the same colour in a peacock's tail gold. Another that hath better observed, adds to shining yellow great weight: and then the sound gold, when he uses it, stands for a complex idea of a shining yellow and a very weighty substance. Another adds to those qualities fusibility: and then the word gold signifies to him a body, bright, yellow, fusible, and very heavy. Another adds malleability. Each of these uses equally the word gold, when they have occasion to express the idea which they have applied it to: but it is evident that each can apply it only to his own idea; nor can he make it stand as a sign of such a complex idea as he has not...

Chapter 3: Of General Terms

1. *The greatest part of words are general terms*. All things that exist being particulars, it may perhaps be thought reasonable that words, which ought to be conformed to things, should be so too, I mean in their signification: but yet we find quite the contrary. The far greatest part of words that make all languages are general terms: which has not been the effect of neglect or chance, but of reason and necessity.

2. *That every particular thing should have a name for itself is impossible*. First, It is impossible that every particular thing should have a distinct peculiar name. For, the signification and use of words depending on that connexion which the mind makes between its ideas and the sounds it uses as signs of them, it is necessary, in the application of names to things, that the mind should have distinct ideas of the things, and retain also the particular name that belongs to every one, with its peculiar appropriation to that idea. But it is beyond the power of human capacity to frame and retain distinct ideas of all the particular things we meet with: every bird and beast men saw; every tree and plant that affected the senses, could not find a place in the most capacious understanding. If it be looked on as an instance of a prodigious memory, that some generals have been able to call every soldier in their army by his proper name, we may easily find a reason why men have never attempted to give names to each sheep in their flock, or crow that flies over their heads; much less to call every leaf of plants, or grain of sand that came in their way, by a peculiar name.

3. *And would be useless, if it were possible*. Secondly, If it were possible, it would yet be useless; because it would not serve to the chief end of language. Men would in vain heap up names of particular things, that would not serve them to communicate their thoughts. Men learn names, and use them in talk with others, only that they may be understood: which is then only done when, by use or consent, the sound I make by the organs of speech, excites in another man's mind who hears it, the idea I apply it to in mine, when I speak it. This cannot be done by names applied to particular things; whereof I alone having the ideas in my mind, the names of them could not be significant or intelligible to another, who was not acquainted with all those very particular things which had fallen under my notice.

4. *A distinct name for every particular thing, not fitted for enlargement of knowledge*. Thirdly, But yet, granting this also feasible, (which I think is not), yet a distinct name for every particular thing would not be of any great use for the improvement of knowledge: which, though founded in particular things, enlarges itself by general views; to which things reduced into sorts, under general names, are properly subservient. These, with the names belonging to them, come within some compass, and do not multiply every moment, beyond what either the mind can contain, or use requires. And therefore, in these, men have for the most part stopped: but yet not so as to hinder themselves from distinguishing particular things by appropriated names, where convenience demands it. And therefore in their own species, which they have most to do with, and wherein they have often occasion to mention particular persons, they make use of proper names; and there distinct individuals have distinct denominations.

5. *What things have proper names, and why*. Besides persons, countries also, cities, rivers, mountains, and other the like distinctions of place have usually found peculiar names, and that for the same reason; they being such as men have often an occasion to mark particularly, and, as it were, set before others in their discourses with them. And I doubt not but, if we had reason to mention particular horses as often as we have to mention particular men, we should have proper names for the one, as familiar as for the other, and Bucephalus would be a word as much in use as Alexander. And therefore we see that, amongst jockeys, horses have their proper names to be known and distinguished by, as commonly as their servants: because, amongst them, there is often occasion to mention this or that particular horse when he is out of sight.

6. *How general words are made*. The next thing to be considered is how general words come to be made. For, since all things that exist are only particulars, how come we by general terms; or where find we those general natures they are supposed to stand for? Words become general by being made the signs of general ideas: and ideas become general, by separating from them the circumstances of time and place, and any other ideas that may determine them to this or that particular existence. By this way of abstraction they are made capable of representing more individuals than one; each of which having in it a conformity to that abstract idea, is (as we call it) of that sort.

7. *Shown by the way we enlarge our complex ideas from infancy*. But, to deduce this a little more distinctly, it will not perhaps be amiss to trace our notions and names from their beginning, and observe by what degrees we proceed, and by what steps we enlarge our ideas from our first infancy. There is nothing more evident, than that the ideas of the persons children converse with (to instance in them alone) are, like the persons themselves, only particular. The ideas of the nurse and the mother are well framed in their minds; and, like pictures of them there, represent only those individuals. The names they first gave to them are confined to these individuals; and the names of nurse and mamma, the child uses, determine themselves to those persons. Afterwards, when time and a larger acquaintance have made them observe that there are a great many other things in the world, that in some common agreements of shape, and several other qualities, resemble their father and mother, and those persons they have been used to, they frame an idea, which they find those many particulars do partake in; and to that they give, with others, the name man, for example. And thus they come to have a general name, and a general idea. Wherein they make nothing new; but only leave out of the complex idea they had of Peter and James, Mary and Jane, that which is peculiar to each, and retain only what is common to them all.

8. *And further enlarge our complex ideas, by still leaving out properties contained in them*. By the same way that they come by the general name and idea of man, they easily advance to more general names and notions. For, observing that several things that differ from their idea of man, and cannot therefore be comprehended under that name, have yet certain qualities wherein they agree with man, by retaining only those qualities, and uniting them into one idea, they have again another and more general idea; to which having given a name they make a term of a more comprehensive extension: which new idea is made, not by any new addition, but only as before, by leaving out the shape, and some other properties signified by the name man, and retaining only a body, with life, sense, and spontaneous motion, comprehended under the name animal....

12. *Abstract ideas are the essences of genera and species*. The next thing therefore to be considered is, What kind of signification it is that general words have. For, as it is evident that they do not signify barely one particular thing; for then they would not be general terms, but proper names, so, on the other side, it is as evident they do not signify a plurality; for man and men would then signify the same; and the distinction of numbers (as the grammarians call them) would be superfluous and useless. That then which general words signify is a sort of thing; and each of them does that, by being a sign of an abstract idea in the mind; to which idea, as things existing are found to agree, so they come to be ranked under that name, or, which is all one, be of that sort. Whereby it is evident that the essences of the sorts, or, if the Latin word pleases better, species of things, are nothing else but these abstract ideas. For the having the essence of any species, being that which makes anything to be of that species; and the conformity to the idea to which the name is annexed being that which gives a right to that name; the having the essence, and the having that conformity, must needs be the same thing: since to be of any species, and to have a right to the name of that species, is all one. As, for example, to be a man, or of the species man, and to have right to the name man, is the same thing. Again, to be a man, or of the species man, and have the essence of a man, is the same thing. Now, since nothing can be a man, or have a right to the name man, but what has a conformity to the abstract idea the name man stands for, nor anything be a man, or have a right to the species man, but what has the essence of that species; it follows, that the abstract idea for which the name stands, and the essence of the species, is one and the same. From whence it is easy to observe, that the

essences of the sorts of things, and, consequently, the sorting of things, is the workmanship of the understanding that abstracts and makes those general ideas.

§II: On Intuitive and Demonstrative Knowledge
Book IV, Chapter 2: Of the Degrees of Our Knowledge

1. *Of the degrees, or differences in clearness, of our knowledge*. 1. Intuitive. All our knowledge consisting, as I have said, in the view the mind has of its own ideas, which is the utmost light and greatest certainty we, with our faculties, and in our way of knowledge, are capable of, it may not be amiss to consider a little the degrees of its evidence. The different clearness of our knowledge seems to me to lie in the different way of perception the mind has of the agreement or disagreement of any of its ideas. For if we will reflect on our own ways of thinking, we will find, that sometimes the mind perceives the agreement or disagreement of two ideas immediately by themselves, without the intervention of any other: and this I think we may call intuitive knowledge. For in this the mind is at no pains of proving or examining, but perceives the truth as the eye doth light, only by being directed towards it. Thus the mind perceives that white is not black, that a circle is not a triangle, that three are more than two and equal to one and two. Such kinds of truths the mind perceives at the first sight of the ideas together, by bare intuition; without the intervention of any other idea: and this kind of knowledge is the clearest and most certain that human frailty is capable of. This part of knowledge is irresistible, and, like bright sunshine, forces itself immediately to be perceived, as soon as ever the mind turns its view that way; and leaves no room for hesitation, doubt, or examination, but the mind is presently filled with the clear light of it. It is on this intuition that depends all the certainty and evidence of all our knowledge; Which certainty every one finds to be so great, that he cannot imagine, and therefore not require a greater: for a man cannot conceive himself capable of a greater certainty than to know that any idea in his mind is such as he perceives it to be; and that two ideas, wherein he perceives a difference, are different and not precisely the same. He that demands a greater certainty than this, demands he knows not what, and shows only that he has a mind to be a sceptic, without being able to be so. Certainty depends so wholly on this intuition, that, in the next degree of knowledge which I call demonstrative, this intuition is necessary in all the connexions of the intermediate ideas, without which we cannot attain knowledge and certainty.

2. *2. Demonstrative*. The next degree of knowledge is, where the mind perceives the agreement or disagreement of any ideas, but not immediately. Though wherever the mind perceives the agreement or disagreement of any of its ideas, there be certain knowledge; yet it does not always happen, that the mind sees that agreement or disagreement, which there is between them, even where it is discoverable; and in that case remains in ignorance, and at most gets no further than a probable conjecture. The reason why the mind cannot always perceive presently the agreement or disagreement of two ideas, is, because those ideas, concerning whose agreement or disagreement the inquiry is made, cannot by the mind be so put together as to show it. In this case then, when the mind cannot so bring its ideas together as by their immediate comparison, and as it were juxtaposition or application one to another, to perceive their agreement or disagreement, it is fain, by the intervention of other ideas (one or more, as it happens) to discover the agreement or disagreement which it searches; and this is that which we call reasoning. Thus, the mind being willing to know the agreement or disagreement in bigness between the three angles of a triangle and two right ones, cannot by an immediate view and comparing them do it: because the three angles of a triangle cannot be brought at once, and be compared with any other one, or two, angles; and so of this the mind has no immediate, no intuitive knowledge. In this case the mind is fain to find out some other angles, to which the three angles of a triangle have an equality; and, finding those equal to two right ones. comes to know their equality to two right ones.

3. *Demonstration depends on clearly perceived proofs*. Those intervening ideas, which serve to show the agreement of any two others, are called proofs; and where the agreement and disagreement is by this means plainly and clearly perceived, it is called demonstration; it being shown to the understanding, and the mind made to see that it is so. A quickness in the mind to find out these intermediate ideas, (that shall discover the agreement or disagreement of any other,) and to apply them right, is, I suppose, that which is called sagacity.

4. *As certain, but not so easy and ready as intuitive knowledge*. This knowledge, by intervening proofs, though it be certain, yet the evidence of it is not altogether so clear and bright, nor the assent so ready, as in intuitive knowledge. For, though in demonstration the mind does at last perceive the agreement or disagreement of the ideas it considers; yet it is not without pains and attention: there must be more than one transient view to find it. A steady application and pursuit are required to this discovery: and there must be a progression by steps and degrees, before the mind can

in this way arrive at certainty, and come to perceive the agreement or repugnancy between two ideas that need proofs and the use of reason to show it.

5. *The demonstrated conclusion not without doubt, precedent to the demonstration*. Another difference between intuitive and demonstrative knowledge is, that, though in the latter all doubt be removed when, by the intervention of the intermediate ideas, the agreement or disagreement is perceived, yet before the demonstration there was a doubt; which in intuitive knowledge cannot happen to the mind that has its faculty of perception left to a degree capable of distinct ideas; no more than it can be a doubt to the eye (that can distinctly see white and black), Whether this ink and this paper be all of a colour. If there be sight in the eyes, it will, at first glimpse, without hesitation, perceive the words printed on this paper different from the colour of the paper: and so if the mind have the faculty of distinct perception, it will perceive the agreement or disagreement of those ideas that produce intuitive knowledge. If the eyes have lost the faculty of seeing, or the mind of perceiving, we in vain inquire after the quickness of sight in one, or clearness of perception in the other.

6. *Not so clear as intuitive knowledge*. It is true, the perception produced by demonstration is also very clear; yet it is often with a great abatement of that evident luster and full assurance that always accompany that which I call intuitive: like a face reflected by several mirrors one to another, where, as long as it retains the similitude and agreement with the object, it produces a knowledge; but it is still, in every successive reflection, with a lessening of that perfect clearness and distinctness which is in the first; till at last, after many removes, it has a great mixture of dimness, and is not at first sight so knowable, especially to weak eyes. Thus it is with knowledge made out by a long train of proof.

7. *Each step in demonstrated knowledge must have intuitive evidence*. Now, in every step reason makes in demonstrative knowledge, there is an intuitive knowledge of that agreement or disagreement it seeks with the next intermediate idea which it uses as a proof: for if it were not so, that yet would need a proof; since without the perception of such agreement or disagreement, there is no knowledge produced: if it be perceived by itself, it is intuitive knowledge: if it cannot be perceived by itself, there is need of some intervening idea, as a common measure, to show their agreement or disagreement. By which it is plain that every step in reasoning that produces knowledge, has intuitive certainty; which when the mind perceives, there is no more required but to remember it, to make the agreement or disagreement of the ideas concerning which we inquire visible and certain. So that to make anything a demonstration, it is necessary to perceive the immediate agreement of the intervening ideas, whereby the agreement or disagreement of the two ideas under examination (whereof the one is always the first, and the other the last in the account) is found. This intuitive perception of the agreement or disagreement of the intermediate ideas, in each step and progression of the demonstration, must also be carried exactly in the mind, and a man must be sure that no part is left out: which, because in long deductions, and the use of many proofs, the memory does not always so readily and exactly retain; therefore it comes to pass, that this is more imperfect than intuitive knowledge, and men embrace often falsehood for demonstrations...

9. *Demonstration not limited to ideas of mathematical quantity*. It has been generally taken for granted, that mathematics alone are capable of demonstrative certainty: but to have such an agreement or disagreement as may intuitively be perceived, being, as I imagine, not the privilege of the ideas of number, extension, and figure alone, it may possibly be the want of due method and application in us, and not of sufficient evidence in things, that demonstration has been thought to have so little to do in other parts of knowledge, and been scarce so much as aimed at by any but mathematicians. For whatever ideas we have wherein the mind can perceive the immediate agreement or disagreement that is between them, there the mind is capable of intuitive knowledge; and where it can perceive the agreement or disagreement of any two ideas, by an intuitive perception of the agreement or disagreement they have with any intermediate ideas, there the mind is capable of demonstration: which is not limited to ideas of extension, figure, number, and their modes.

10. *Why it has been thought to be so limited*. The reason why it has been generally sought for, and supposed to be only in those, I imagine has been, not only the general usefulness of those sciences: but because, in comparing their equality or excess, the modes of numbers have every the least difference very clear and perceivable: and though in extension every the least excess is not so perceptible, yet the mind has found out ways to examine, and discover demonstratively, the just equality of two angles, or extensions, or figures: and both these, i.e. numbers and figures, can be set down by visible and lasting marks, wherein the ideas under consideration are perfectly determined; which for the most part they are not, where they are marked only by names and words.

11. *Modes of qualities not demonstrable like modes of quantity*. But in other simple ideas, whose modes and differences are made and counted by degrees, and not

quantity, we have not so nice and accurate a distinction of their differences as to perceive, or find ways to measure, their just equality, or the least differences. For those other simple ideas, being appearances of sensations produced in us, by the size, figure, number, and motion of minute corpuscles singly insensible; their different degrees also depend upon the variation of some or of all those causes: which, since it cannot be observed by us, in particles of matter whereof each is too subtile to be perceived, it is impossible for us to have any exact measures of the different degrees of these simple ideas. For, supposing the sensation or idea we name whiteness be produced in us by a certain number of globules, which, having a verticity about their own centres, strike upon the retina of the eye, with a certain degree of rotation, as well as progressive swiftness; it will hence easily follow, that the more the superficial parts of any body are so ordered as to reflect the greater number of globules of light, and to give them the proper rotation, which is fit to produce this sensation of white in us, the more white will that body appear, that from an equal space sends to the retina the greater number of such corpuscles, with that peculiar sort of motion. I do not say that the nature of light consists in very small round globules; nor of whiteness in such a texture of parts as gives a certain rotation to these globules when it reflects them: for I am not now treating physically of light or colours. But this I think I may say, that I cannot (and I would be glad any one would make intelligible that he did), conceive how bodies without us can any ways affect our senses, but by the immediate contact of the sensible bodies themselves, as in tasting and feeling, or the impulse of some sensible particles coming from them, as in seeing, hearing, and smelling; by the different impulse of which parts, caused by their different size, figure, and motion, the variety of sensations is produced in us.

12. *Particles of light and simple ideas of colour*. Whether then they be globules or no; or whether they have a verticity about their own centres that produces the idea of whiteness in us; this is certain, that the more particles of light are reflected from a body, fitted to give them that peculiar motion which produces the sensation of whiteness in us (and possibly too, the quicker that peculiar motion is) the whiter does the body appear from which the greatest number are reflected, as is evident in the same piece of paper put in the sunbeams, in the shade, and in a dark hole; in each of which it will produce in us the idea of whiteness in far different degrees.

13. *The secondary qualities of things not discovered by demonstration*. Not knowing, therefore, what number of particles, nor what motion of them, is fit to produce any precise degree of whiteness, we cannot demonstrate the certain equality of any two degrees of whiteness; because we have no certain standard to measure them by, nor means to distinguish every the least real difference, the only help we have being from our senses, which in this point fail us. But where the difference is so great as to produce in the mind clearly distinct ideas, whose differences can be perfectly retained, there these ideas or colours, as we see in different kinds, as blue and red, are as capable of demonstration as ideas of number and extension. What I have here said of whiteness and colours, I think holds true in all secondary qualities and their modes.

14. *Sensitive knowledge of the particular existence of finite beings without us*. These two, viz. intuition and demonstration, are the degrees of our knowledge; whatever comes short of one of these, with what assurance soever embraced, is but faith or opinion, but not knowledge, at least in all general truths. There is, indeed, another perception of the mind, employed about the particular existence of finite beings without us, which, going beyond bare probability, and yet not reaching perfectly to either of the foregoing degrees of certainty, passes under the name of knowledge. There can be nothing more certain than that the idea we receive from an external object is in our minds: this is intuitive knowledge. But whether there be anything more than barely that idea in our minds; whether we can thence certainly infer the existence of anything without us, which corresponds to that idea, is that whereof some men think there may be a question made; because men may have such ideas in their minds, when no such thing exists, no such object affects their senses. But yet here I think we are provided with an evidence that puts us past doubting. For I ask any one, Whether he be not invincibly conscious to himself of a different perception, when he looks on the sun by day, and thinks on it by night; when he actually tastes wormwood, or smells a rose, or only thinks on that savour or odour? We as plainly find the difference there is between any idea revived in our minds by our own memory, and actually coming into our minds by our senses, as we do between any two distinct ideas. If any one say, a dream may do the same thing, and all these ideas may be produced in us without any external objects; he may please to dream that I make him this answer: 1. That it is no great matter, whether I remove his scruple or no: where all is but dream, reasoning and arguments are of no use, truth and knowledge nothing. 2. That I believe he will allow a very manifest difference between dreaming of being in the fire, and being actually in it. But yet if he be resolved to appear so sceptical as to maintain, that what I call being actually in the fire is nothing but a dream; and that we cannot thereby certainly know, that any such thing as fire actually exists without us: I answer, That we certainly finding

that pleasure or pain follows upon the application of certain objects to us, whose existence we perceive, or dream that we perceive, by our senses; this certainty is as great as our happiness or misery, beyond which we have no concernment to know or to be. So that, I think, we may add to the two former sorts of knowledge this also, of the existence of particular external objects, by that perception and consciousness we have of the actual entrance of ideas from them, and allow these three degrees of knowledge, viz. intuitive, demonstrative, and sensitive: in each of which there are different degrees and ways of evidence and certainty.

15. *Knowledge not always clear, where the ideas that enter into it are clear*. But since our knowledge is founded on and employed about our ideas only, will it not follow from thence that it is conformable to our ideas; and that where our ideas are clear and distinct, or obscure and confused, our knowledge will be so too? To which I answer, No: for our knowledge consisting in the perception of the agreement or disagreement of any two ideas, its clearness or obscurity consists in the clearness or obscurity of that perception, and not in the clearness or obscurity of the ideas themselves: v.g. a man that has as clear ideas of the angles of a triangle, and of equality to two right ones, as any mathematician in the world, may yet have but a very obscure perception of their agreement, and so have but a very obscure knowledge of it. But ideas which, by reason of their obscurity or otherwise, are confused, cannot produce any clear or distinct knowledge; because, as far as any ideas are confused, so far the mind cannot perceive clearly whether they agree or disagree. Or to express the same thing in a way less apt to be misunderstood: he that hath not determined ideas to the words he uses, cannot make propositions of them of whose truth he can be certain.

§III: On Mathematical Knowledge
Book IV, Chapter 4: Of the Reality of Knowledge

1. *Objection. "Knowledge placed in our ideas may be all unreal or chimerical*." I doubt not but my reader, by this time, may be apt to think that I have been all this while only building a castle in the air; and be ready to say to me: "To what purpose all this stir? Knowledge, say you, is only the perception of the agreement or disagreement of our own ideas: but who knows what those ideas may be? Is there anything so extravagant as the imaginations of men's brains? Where is the head that has no chimeras in it? Or if there be a sober and a wise man, what difference will there be, by your rules, between his knowledge and that of the most extravagant fancy in the world? They both have their ideas, and perceive their agreement and disagreement one with another. If there be any difference between them, the advantage will be on the warm-headed man's side, as having the more ideas, and the more lively. And so, by your rules, he will be the more knowing. If it be true, that all knowledge lies only in the perception of the agreement or disagreement of our own ideas, the visions of an enthusiast and the reasonings of a sober man will be equally certain. It is no matter how things are: so a man observe but the agreement of his own imaginations, and talk conformably, it is all truth, all certainty. Such castles in the air will be as strongholds of truth, as the demonstrations of Euclid. That an harpy is not a centaur is by this way as certain knowledge, and as much a truth, as that a square is not a circle."

"But of what use is all this fine knowledge of men's own imaginations, to a man that inquires after the reality of things? It matters not what men's fancies are, it is the knowledge of things that is only to be prized: it is this alone gives a value to our reasonings, and preference to one man's knowledge over another's, that it is of things as they really are, and not of dreams and fancies."

2. *Answer: "Not so, where ideas agree with things."* To which I answer, That if our knowledge of our ideas terminate in them, and reach no further, where there is something further intended, our most serious thoughts will be of little more use than the reveries of a crazy brain; and the truths built thereon of no more weight than the discourses of a man who sees things clearly in a dream, and with great assurance utters them. But I hope, before I have done, to make it evident, that this way of certainty, by the knowledge of our own ideas, goes a little further than bare imagination: and I believe it will appear that all the certainty of general truths a man has lies in nothing else.

3. *But what shall be the criterion of this agreement?* It is evident the mind knows not things immediately, but only by the intervention of the ideas it has of them. Our knowledge, therefore is real only so far as there is a conformity between our ideas and the reality of things. But what shall be here the criterion? How shall the mind, when it perceives nothing but its own ideas, know that they agree with things themselves? This, though it seems not to want difficulty, yet, I think, there be two sorts of ideas that we may be assured agree with things.

4. *As all simple ideas are really conformed to things*. First, The first are simple ideas, which since the mind, as has been shown, can by no means make to itself, must necessarily be the product of things operating on the mind, in a natural way, and producing therein those perceptions which by the Wisdom and Will of our

Maker they are ordained and adapted to. From whence it follows, that simple ideas are not fictions of our fancies, but the natural and regular productions of things without us, really operating upon us; and so carry with them all the conformity which is intended; or which our state requires: for they represent to us things under those appearances which they are fitted to produce in us: whereby we are enabled to distinguish the sorts of particular substances, to discern the states they are in, and so to take them for our necessities, and apply them to our uses. Thus the idea of whiteness, or bitterness, as it is in the mind, exactly answering that power which is in any body to produce it there, has all the real conformity it can or ought to have, with things without us. And this conformity between our simple ideas and the existence of things, is sufficient for real knowledge.

5. *All complex ideas, except ideas of substances, are their own archetypes*. Secondly, All our complex ideas, except those of substances, being archetypes of the mind's own making, not intended to be the copies of anything, nor referred to the existence of anything, as to their originals, cannot want any conformity necessary to real knowledge. For that which is not designed to represent anything but itself, can never be capable of a wrong representation, nor mislead us from the true apprehension of anything, by its dislikeness to it: and such, excepting those of substances, are all our complex ideas. Which, as I have shown in another place, are combinations of ideas, which the mind, by its free choice, puts together, without considering any connexion they have in nature. And hence it is, that in all these sorts the ideas themselves are considered as the archetypes, and things no otherwise regarded, but as they are conformable to them. So that we cannot but be infallibly certain, that all the knowledge we attain concerning these ideas is real, and reaches things themselves. Because in all our thoughts, reasonings, and discourses of this kind, we intend things no further than as they are conformable to our ideas. So that in these we cannot miss of a certain and undoubted reality.

6. *Hence the reality of mathematical knowledge*. I doubt not but it will be easily granted, that the knowledge we have of mathematical truths is not only certain, but real knowledge; and not the bare empty vision of vain, insignificant chimeras of the brain: and yet, if we will consider, we shall find that it is only of our own ideas. The mathematician considers the truth and properties belonging to a rectangle or circle only as they are in ideas in his own mind. For it is possible he never found either of them existing mathematically, i.e. precisely true, in his life. But yet the knowledge he has of any truths or properties belonging to a circle, or any other mathematical figure, are nevertheless true and certain, even of real things existing: because real things are no further concerned, nor intended to be meant by any such propositions, than as things really agree to those archetypes in his mind. Is it true of the idea of a triangle, that its three angles are equal to two right ones? It is true also of a triangle, wherever it really exists. Whatever other figure exists, that it is not exactly answerable to that idea of a triangle in his mind, is not at all concerned in that proposition. And therefore he is certain all his knowledge concerning such ideas is real knowledge: because, intending things no further than they agree with those his ideas, he is sure what he knows concerning those figures, when they have barely an ideal existence in his mind, will hold true of them also when they have a real existence in matter: his consideration being barely of those figures, which are the same wherever or however they exist.

7. *And of moral*. And hence it follows that moral knowledge is as capable of real certainty as mathematics. For certainty being but the perception of the agreement or disagreement of our ideas, and demonstration nothing but the perception of such agreement, by the intervention of other ideas or mediums; our moral ideas, as well as mathematical, being archetypes themselves, and so adequate and complete ideas; all the agreement or disagreement which we shall find in them will produce real knowledge, as well as in mathematical figures.